

SPHERICAL GEOMETRY











































HSC

Name: _____

HSC CAPACITY MATRIX - GENERAL MATHEMATICS

TOPIC: Measurement 7 - Spherical Geometry

2 weeks

CONTENT	CAPACITY BREAKDOWN!	DONE IT!!!!	GOT IT!!!!	ON MY WAY!	WORKING ON IT!	HELP!!!!
1. Calculating arc lengths of circles	Ex 13A Q1a, c, e, 2, 3a, c, e, 4-7, 8a, c, e, 9-13					
2. Distinguishing between great and small circles	Ex 13B					
3. Using the Equator and the Greenwich Meridian as lines of reference for locations on the Earth's surface	Ex 13C					
4. Locating positions on the globe using latitude and longitude						
5. Converting nautical miles to kilometres and vice versa	Ex 13D Credit card research task					
6. Calculating distances between 2 points on the same great circle in nautical miles and km						
7. Defining 1 knot as a speed of 1nM per hour	Ex 13E Where in the World are the Simpsons? task					
8. Using time zones and the International Date line in solving problems						
9. Calculating time differences given the difference in longitudes						
10. Determining times for cities in different countries in related travel questions						

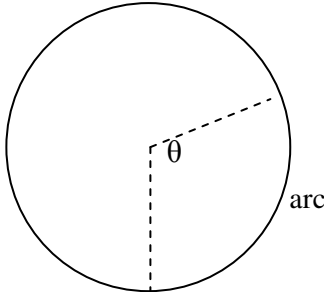
Your say!

What was the most important thing you learned? _____

What was something new you learnt? _____

What part(s) of this topic will you need to work on? _____

ARC LENGTH



An **arc** is part of the circumference of a circle

A **major arc** is more than half of the circumference and a **minor arc** is less than half of the circumference.

The length l , of an arc in a circle with radius r can be found using:

$$l = \frac{\theta}{360} \times 2\pi r$$

where θ is the angle subtended by the arc at the centre of the circle.

eg The radius of the Earth is approximately 6 400 km.

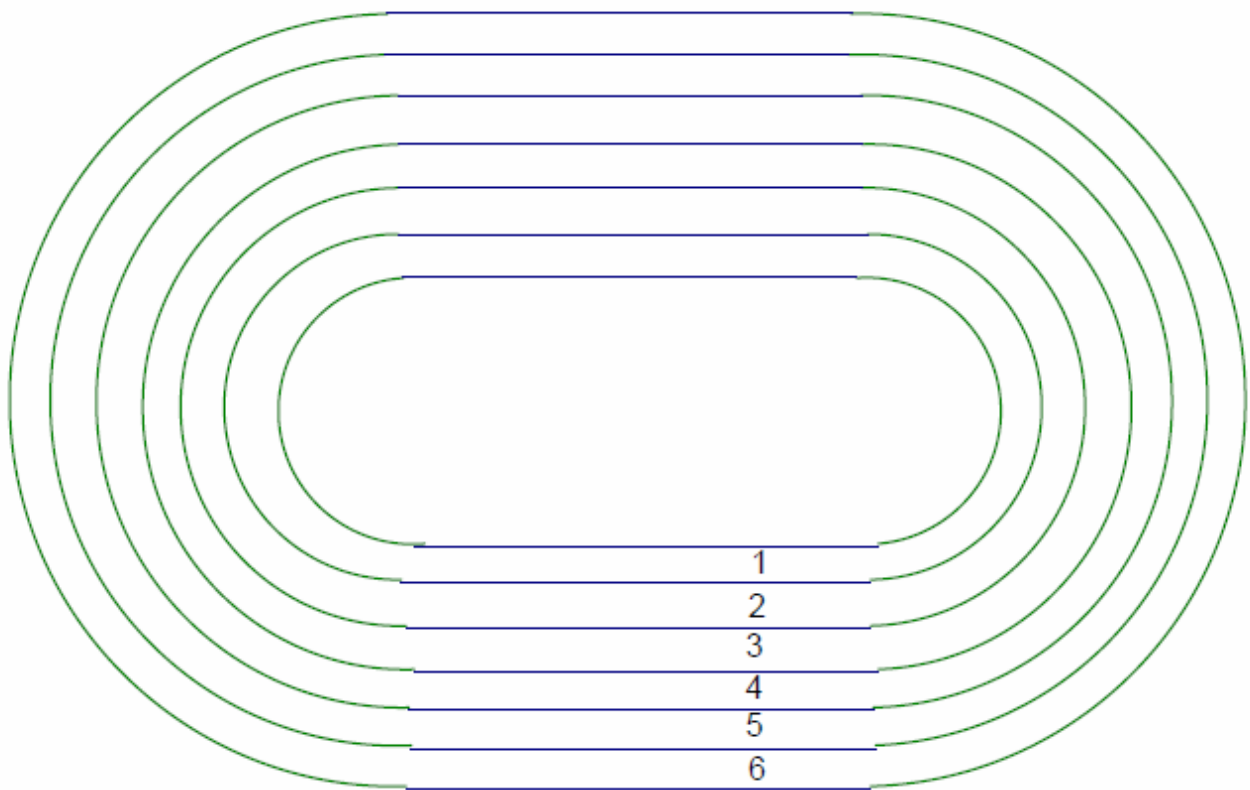
a) Calculate the circumference of the Earth at the equator, correct to the nearest kilometre.

b) Two points of the equator subtend at a 15° angle at the centre of the Earth.
Calculate the distance between them, correct to the nearest kilometre;

c) The distance between two points on the Equator is 17 500km. What is the angular distance between the points?

Application of Arc Length

You are to determine the approximate starting positions for a standard shaped 400m running track which is composed of two congruent straight-aways and two congruent circles as shown below. The width of a lane is 1.2m.

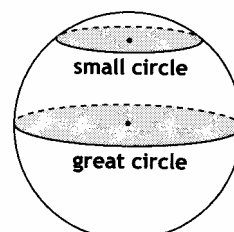


GREAT AND SMALL CIRCLES

The intersection of a plane and a sphere is a circle. [Think about a knife (a plane) slicing through an orange (a sphere) – the result is a circle]

A **great circle** is formed when a plane cuts the surface of a sphere and passes through its centre.

A **small circle** is formed when a plane cuts the surface of a sphere and does not pass through its centre.



Some of the important features of Earth are shown in the following

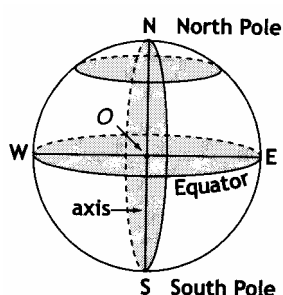


diagram:

Antipodal points are the endpoints of a diameter of a great circle. N, S and W, E are two pairs of antipodal points.

NOS is the **axis** of Earth.

N and S are called the poles.

The great circle WE is the **Equator**.

The angle between two great circles is the angle between their planes.

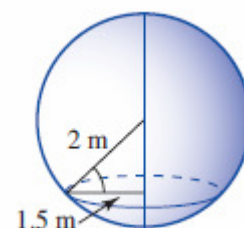
The **length of a great circle** can be calculated by

$$C = 2\pi r \text{ OR } C = \pi D$$

To calculate the length around a small circle, the radius of the small circle must be known.

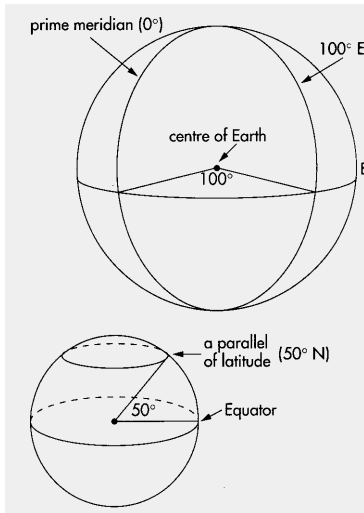
eg Calculate the length of a great circle on a sphere with a radius of 55cm. Give your answer correct to the nearest centimetre.

eg Calculate the length of the small circle from the diagram, correct to 1 decimal place.



LATITUDE & LONGITUDE

Parallels of Latitude are small circles that are parallel to the Equator, latitude 0° . They are marked off in degrees from the equator so latitude is the angular distance of a point **NORTH** or **SOUTH** of the equator.



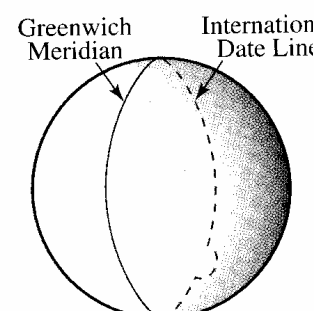
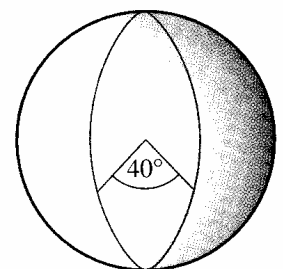
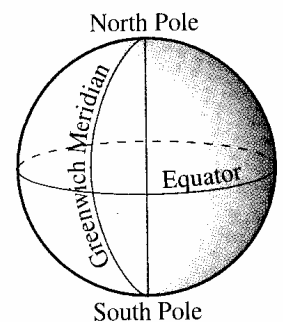
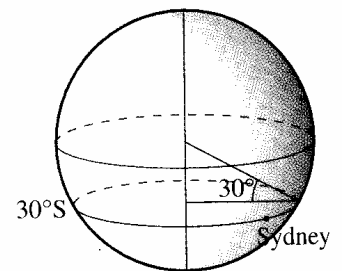
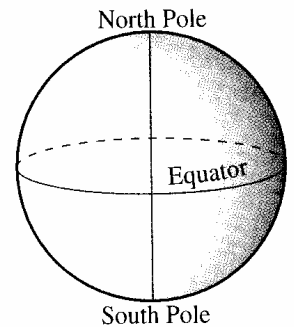
Meridians of Longitude are great circles that pass through the north and south poles. Longitude is measured in degrees **EAST** of **WEST** of the Greenwich meridian, longitude 0° (which runs through Greenwich, a suburb of London England)

The meridian of longitude opposite

Greenwich is the **International Date Line**. It has longitude 180° either east or west. On either side of the International Date Line the day changes. For this reason, the International Date Line does not pass through any land mass.

The **position** of any point on the surface of the Earth can be specified by using latitude and longitude, in that order! The 2 values are written as an ordered pair, eg the coordinates of Sydney are 30°S , 150°E .

The **nautical mile** (n mile) is equal to **1 852 km** and is used in navigation to measure the distance travelled on the surface of the Earth. It is defined as the length of an arc of a great circle subtended by $1''$ (1 minute: 60 minutes = 1°)





THINK!

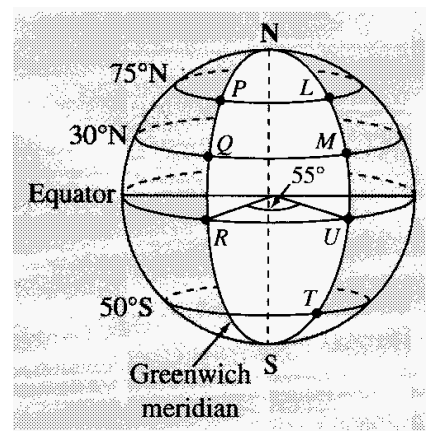
LONGitude always has LONG lines.

LATitude rymes with SAT - so you can sit on lines of latitude.

The conversion for nm is simply the middle buttons of your calculator, from top to bottom.

eg From the given diagram:

a) Give the locations of points P, Q, T and L using latitude and longitude.



b) Which point is at (30°N, 55°E) and which point is a (0°, 55°E)?

c) Using the map overleaf, identify the major cities closest to the following locations:

(i) 30°S, 30°E _____

(ii) 30°N, 120°E _____

(iii) 45°N, 75°W _____

d) Select three cities from the map that you would like to visit and write the name and coordinates:

(i) _____

(ii) _____

(iii) _____

DISTANCES ON THE EARTH'S SURFACE

POINTS TO CONSIDER:

- 🌐 The angular distance between 2 points on the same meridian of longitude or the equator can be found by:
 - 🔧 **SUBTRACTING** the latitudes if the points are on the SAME side of the equator;
 - 🔧 **ADDING** the latitudes if the points are on opposite sides of the equator.
- 🌐 An angular distance can be converted to a distance using $1^\circ = 60 \text{ nm}$
- 🌐 The distance between 2 points on the same great circle can also be found using the arc length formula.
- 🌐 Speed can be measured in knots. A speed of **1 knot = 1nm/h**
- 🌐 The distance along a small circle can be found by

$$r = R \cos \alpha \text{ where}$$

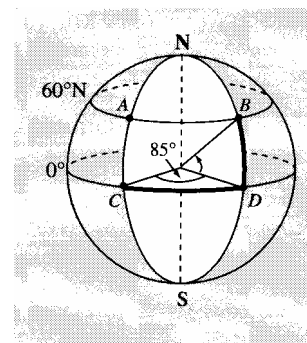
r is the radius of small circle

R is the radius of great circle

α is the latitude of the small circle

eg A great circle arc CD makes an angle of 85° at the centre of the Earth.

- a) Find the spherical distance CD in nautical miles.
- b) Calculate the distance between points B and D in nautical miles.



eg Find the distance between the point A 60°N , 50°E and the point B 60°N , 170°E to the nearest kilometre.

1. Draw a 3D picture;
2. Calculate the radius of the small circle;
3. Calculate the arc length
4. State the answer



eg A ship travels through an arc of 22° around the 30°S parallel of latitude.

- a) Calculate the distance travelled, correct to the nearest nautical mile.
- b) Calculate the average speed of the ship, in knots, if the journey took 77 hours.

eg A ship travels from 120°E to $135^\circ 40'\text{E}$ along the Equator.

- a) Calculate the distance travelled in nautical miles.



- b) Calculate the time taken for the ship to sail between these two points at an average speed of 50 knots.

TIME ZONES & STANDARD TIMES

AUSTRALIAN TIME ZONES

Australia is divided into 3 time zones:

- ④ Eastern Standard Time – NSW, Queensland, Victoria, Tasmania
- ④ Central Standard Time (-0.5h) – NT, South Australia
- ④ Western Standard Time (-2 h) – Western Australia
- ④ If a state has Daylight saving, then ONE HOUR has been added (so to calculate the time against it, subtract 1 hour)

eg Merimbula is GMT +10 while New York is GMT -5.

- a) Calculate the time difference between Merimbula and New York;
- b) We currently have daylight saving, so if it is Tuesday 11:00 am in Merimbula, what is the actual time in New York;

eg Adelaide is GMT +9.5 while Athens is GMT +1. When it is 11:00am in Athens, what is the time in Adelaide?

TIME CONVERSIONS

The time calculated by the longitude is called standard time. Time zones are calculated to approximate all the standard times in a region.

As there are 24 hours in a day and 360° of longitude, we can calculate that:

$$1 \text{ hour} = \text{___}^\circ \text{ of longitude;}$$

$$1^\circ = \text{___} \text{ minutes}$$

- ④ When we move EAST across the IDL we put the date back one day;
- ④ When we move WEST across the IDL we add a day;

④ Dividing the degrees by 15 gives the number of hours difference;

eg Great Aunt Edna wants to watch the Ashes series but she is currently holidaying in London. If the match begins on Thursday, 10:00am in Brisbane 150°E , what time will Edna need to turn on the TV?

eg In Los Angeles 120°W a tennis match was held at 9:00pm on Tuesday. This match was televised live in Sydney. On what day and what time (Eastern Standard time) was the match shown on Sydney television?

eg A (50°S , 20°W) and B (50°S , 60°W) are two points on Earth's surface. The radius of Earth is approximately 6400 km.

- a) Neatly sketch the above information;
- b) Calculate the local time at A if it is 9:00am in B.
- c) Calculate the radius of the parallel of latitude 50°S to the nearest kilometre.
- d) Calculate the shortest distance between A and B along the parallel of latitude 50°S .

WHERE IN THE WORLD ARE THE SIMPSONS!!!



Web sites to use:

[World Time Differences](http://www.whitepages.com.au/wp/search/tools.jhtml) <http://www.whitepages.com.au/wp/search/tools.jhtml>

[World Atlas site](http://worldatlas.com/aatlas/world.htm) <http://worldatlas.com/aatlas/world.htm>









TASK: The Simpsons have gone on a trip to Australia. As always, the family do not follow instructions and now Marge has lost them all. Thankfully everyone has a mobile phone so she can contact them.

To try and organise the mess, Marge has decided to create a chart so she knows when to ring everyone. It is very important that both 12 hour time and 24 hour time is given, plus the day.

Also Marge has decided to mark on a world map where each person (and dog) are for reference.

You have offered to help Marge out and complete the chart and map.



MARGE IS IN SYDNEY, AUSTRALIA	 BART	 MAGGIE	 LISA	 HOMER	 MOE	 SANTA'S LITTLE HELPER	 KRUSTY	 GRANDPA
IS IN...	PRISON III NEW YORK, USA	CHILD CARE III PERTH, AUSTRALIA	THE LOUVRE III PARIS, FRANCE	A TEA HOUSE III TOKYO, JAPAN	A TAVERN III ATHENS, GREECE	A FOX HUNT III KENT, ENGLAND	A CARNIVAL III RIO, ARGENTINA	THE CALGARY STAMPEDE, CANADA
IF MARGE RINGS AT 9:00am ON A MONDAY								
IF MARGE RINGS AT 5:30pm ON A FRIDAY								
EACH PERSON CAN ONLY BE CONTACTED AT 11:00am AT THEIR LOCAL TIME	MARGE WILL NEED TO RING AT	MARGE WILL NEED TO RING AT	MARGE WILL NEED TO RING AT	MARGE WILL NEED TO RING AT	MARGE WILL NEED TO RING AT	MARGE WILL NEED TO RING AT	MARGE WILL NEED TO RING AT	MARGE WILL NEED TO RING AT
ARE THEY BEHIND OR AHEAD III TIME TO MARGE?								
WHAT IS THE TIME DIFFERENCE								
MARGE HAS ORGANISED LAWYERS TO HELP OUT. EVERY LAWYER WILL RING FROM SYDNEY AT 2:45 pm on a Tuesday. WHAT TIME WILL EACH EXPECT THE CALL?								



WHERE IN THE WORLD ARE THE
SIMPSONS MAP